First Homework, due Monday September 14, 2009

Please solve problems 1, 4, 5, 8 on page 85-87 of L.C.Evans' book.

Problem 5: Let $U \subset \mathcal{R}^3$ be open and $u(x) \in C^2(U)$ satisfy the equation

$$\Delta u - \mu^2 u = 0 \ ,$$

where $\mu > 0$. Show that for any ball $B_r(x) \subset U$

$$u(x) = \frac{\mu r}{\sinh(\mu r)} \int_{\partial B_r(x)} u(y) dS(y) \ .$$

Hint: Show that

$$\mathrm{div} \left[\frac{\sinh(\mu|x|)}{\mu|x|} \nabla u(x) - u(x) \nabla \frac{\sinh(\mu|x|)}{\mu|x|} \right] = 0$$

and integrate this identity over the ball $B_r(x)$.