Second Homework, due Wednesday October 7, 2009

1) The initial temperature distribution of a rod of length is given by

$$Ax(1-x)\ ,\ 0 \leq x \leq 1\ ,$$

where A is a constant. Find the temperature distribution at time t when both ends of the rod are kept at zero temperature.

2) Let U be a bounded open and smooth domain in $\mathcal{R}^n$. Consider $u(x,t)$ the solution of the initial value problem

$$u_t = \Delta u \quad , \text{ in } U \times (0,\infty)$$

$$u = f \quad , \text{ on } U \times \{t=0\}$$

$$u = 0 \quad , \text{ on } \partial U \times (0,\infty)\ .$$

Let $\Gamma \subset \partial U$ be a portion of the boundary. Calculate the total amount of heat that flows across Γ. Express your answer in terms of the *harmonic measure* given by

$$\Delta h = 0 \quad , \text{ in } U$$

$$h = 1 \quad , \text{ on } \Gamma$$

$$h = 0 \quad , \text{ on } \partial U \setminus \Gamma\ .$$

3) Please solve Problems 11 and 16 on page 87/88 of L.C Evans' book.

4) By descending from two to one dimension, proof d'Alembert's formula for the initial value problem

$$u_{tt} - u_{xx} = 0 \quad , \text{ on } \mathcal{R} \times (0,\infty)$$

$$u = g\ , u_t = h \quad , \text{ in } \mathcal{R} \times \{t=0\}\ .$$