Third Homework, due Wednesday November 4 , 2009

1) Solve the following initial value problems

$$a) \quad x^2 u_x + y^2 u_y = u^2 \ , \quad u(x,1) = \cos x \ .$$

$$b) \quad x u_x + y u_y = u \ , \quad u(x,x^2) = 1 \ , x > 0 \ .$$

c): Solve problem 3 c) on page 163 of L.C. Evans' book.

2) Do problem 6 and 7 on page 163 of L.C. Evans' book.

3) Find all solutions $u(x,t)$ of the equation

$$u_t = F(u_x) \ , \quad u(x,0) = h(x) \ .$$

F and h are given differentiable functions.

4) Solve the equation

$$\rho_t + c(1-2\rho)\rho_x = 0$$

where $\rho(x,0)$ is given by the function $f(x)$ that vanishes for negative x, equals x for x between 0 and 1, and is equals to 1 for $x > 1$. This is the traffic flow problem. Here c is the speed limit of the highway and $\rho(x,t)$ is the normalized density of cars.