## First Homework, due Monday September 10, 2007

1) Consider the PDE

$$u_t + u^2 u_x = 0$$
,  $u(0, x) = g(x)$ .

Are there examples of initial conditions where the solution ceases to exist after a finite time  $t \ge 0$ ? Are there any initial condition for which there are global solutions? Try to find explicitly solvable examples. Verify your solution.

2) Solve the following initial value problems and verify your solution:

a) 
$$u_t + uu_x = 1$$
,  $u(0, x) = x$ .

b) 
$$u_t + u_x = u^2$$
,  $u(0, x) = g(x)$ .

c) 
$$u_t - xuu_x = 0$$
,  $u(0, x) = x$ 

4) Solve

$$u_t + uu_x = -x$$
,  $u(0, x) = 0$ 

Determine the largest time T so that the solution exist for all  $t \in [0, T]$ . What does this mean in terms of the characteristic?

5) (Challenge) Consider

$$u_t + uu_x = -\sin(x) , u(0, x) = 0 .$$

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Determine the largest time T so that the solution exist for all  $t \in [0, T)$ .