Fourth Homework, due Monday December 3 , 2007

1) Do the problems 5, 11, 14 on pages 163/164 and problem 2 on page 233/234 in L.C. Evans' book.

2) Find the unique entropy solution of the conservation law

$$u_t + \frac{1}{2}(u^2)_x = 0$$

with the initial condition $u(x, 0) = g(x)$ where

$$g(x) = \begin{cases} 0 & \text{if } x < -1 \\ (1 - |x|) & \text{if } -1 \le x \le 1 \\ 0 & \text{if } x > 1. \end{cases}$$

3) Solve the following homgenization problem exactly:

$$-[a(\frac{x}{\varepsilon})u^{\varepsilon}_x]_x = 1 \text{ on } (0, 1)$$

with the boundary conditions

$$u^{\varepsilon}(0) = u^{\varepsilon}(1) = 0 \ .$$

Here, a is a smooth, positive function with period 1.

Proceed as follows. Derive, using a multiscale expansion the equation

$$-\overline{a}u_{xx} = 1 \text{ on } (0, 1) \ .$$

Determine $\overline{a}$.

Find the solution of this equation subject to the boundary conditions

$$u(0) = u(1) = 0 \ .$$