Test I for Calculus II, Math 1502, February 1, 2000

Name:

This test is to be taken without graphing calculators and notes of any sorts. Normal calculators are permitted. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work for otherwise credit cannot be given.

I: (25 points) Consider the function

$$\tan^{-1}(x) = \int_0^x \frac{1}{1+t^2}\mathrm{d}t \ .$$

a) Find the first three terms of the Taylor expansion (around 0) of this function.

b) Using these terms compute an approximate value for $\tan^{-1}(1/2)$.

c) Give an estimate on how accurate that value approximates $\tan^{-1}(1/2)$.

II: (25 points) Compute the limits:

a)

$$\lim_{x\to 0}\left[\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right] .$$

b)

$$\lim_{x\to\infty}\frac{e^{\frac{2}{x}}-1}{\frac{1}{x}} .$$

c)

$$\lim_{x\to 0}\left[\frac{\cos(x)-1+x^2/2}{x^4}\right] .$$

III: (25 points) Which of the following series is convergent or divergent?
a)

$$\sum_{k=2}^{\infty} \frac{1}{k \ln(k)} \ .$$

b)

$$\sum_{k=1}^{\infty} \frac{1}{1+k^2} \ .$$

c) Evaluate the series

$$\sum_{k=1}^{\infty} (\frac{3}{4})^k \ .$$

IV: (25 points) Decide whether the following improper integrals exist and compute them if they do exist.

a)

$$\int_0^1 \frac{x}{\sqrt{1-x^2}}\mathrm{d}x \ .$$

b)

$$\int_{1/2}^2 \frac{1}{x\ln(x)}\mathrm{d}x \ .$$

c)

$$\int_0^\infty xe^{-x^2}\mathrm{d}x \ .$$