Practice Quiz IB for Math 1502

I (a) Let $f(y) = e^y$, and let $R_n(y)$ be the remainder in the nth degree Taylor approximation of this function at $y = 0$; i.e.

$$e^y = P_n(y) + R_n(y) \ .$$

Compute a close bound on

$$\left| \int_2^4 \sqrt{x} R_n(1/x^2) \mathrm{d}x \right| \ .$$

Your answer should be a function of n.

(b) Using a Taylor polynomial approximation, find an elementary integral that could be used to compute

$$\int_2^4 \sqrt{x} e^{1/x^2} \mathrm{d}x$$

to an accuracy of 2×10^{-3}. Explain how you know your answer has the required accuracy.

(II.) **(a)** Consider the function

$$f(x) = 1 - \sec(x^2) \ .$$

Using the formula for Taylor polynomials, compute $P_4(x)$.

(b) Consider the function

$$g(x) = \sin^2(x^2) \ .$$

Using the formula for Taylor polynomials, compute $P_4(x)$.

(c) Compute

$$\lim_{x \to 0} \frac{f(x)}{g(x)} \ .$$

(III.) Determine whether the following infinite series are divergent, conditionally convergent, or absolutely convergent.

(a)

$$\sum_{k=2}^{\infty} \frac{(-1)^k 2k}{\sqrt{k^3 + 1}}$$

(a)

$$\sum_{k=2}^{\infty} \frac{(-1)^k 2^k k!}{k^k}$$

(c)

$$\sum_{k=0}^{\infty} \frac{(3k)!}{(k!)((2k)!)}$$

(d)

$$\sum_{k=0}^{\infty} (-1)^k \left(\sqrt{k} - \sqrt{k-1} \right)^k$$