Test II for Calculus II, Math 1502, February 15, 2000

Name:

This test is to be taken without graphing calculators and notes of any sorts. Normal calculators are permitted. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work for otherwise credit cannot be given.

I: (25 points) Determine whether the following series converge or diverge.

a) $$\sum_{k=1}^{\infty} \frac{2^k k!}{(2k)!}$$

b) $$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

c) $$\sum_{k=1}^{\infty} \left(\frac{k^4}{k^4+1} \right)^{k^2}$$

II: (25 points) Determine the interval of convergence for the following power series:

a) $$\sum_{k=1}^{\infty} \frac{k!}{k^k} x^k$$

b) $$\sum_{k=1}^{\infty} \frac{3k^2}{e^k} (x+1)^k$$

c) $$\sum_{k=1}^{\infty} \left(\frac{k}{1+k} \right)^{k^2} x^k$$

III: (30 points) a) Compute the exact value for

$$\sum_{k=1}^{\infty} k \left(\frac{2}{3}\right)^k \ .$$

b) Compute the power series expansion (around 0) for the function

$$\frac{1}{2-3x} \ .$$

What is its interval of convergence?

c) Give upper and lower bounds for

$$\int_0^1 e^{-x^4} \mathrm{d}x \ ,$$

that agree to an accuracy better than 0.1.

IV: (20 points) Solve the following initial value problem:

$$y' + 3y = 1 \ , \quad y(1) = 2 \ .$$