Preptest for Test III, Calculus II, Math1502, March 14, 2000

I: The questions in this problem check your knowledge of some of the definitions.

a) Find the vector that when added to

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \text{ yields } \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}.$$

b) Which of the following transformations from the plane to the plane is linear. Give a simple linear one and a nonlinear one.

$$T(\vec{x}) = \begin{bmatrix} x^2 - y \\ x - y \end{bmatrix}.$$

$$y_1 = ax_1 + bx_2, \quad y_2 = cx_1 + dx_2.$$

c) Find the length of the vector

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

d) Find the angle between two vectors

$$\begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix}, \quad \begin{bmatrix} -1/3 \\ 2/3 \\ -1/3 \end{bmatrix}.$$

II: Find **all** the solution of the following systems of equations. In each case state whether there is exactly one solution, no solution or infinitely many solutions.

$$a) \quad \begin{aligned} x + 2y + 3z &= 6 \\ x + y &= 2 \\ x - y + 2z &= 2 \end{aligned}$$

$$x + 2y + 3z = 6$$
$$b) \qquad x + y = 2$$
$$y + 3z = 4$$

$$x - y + z = 1$$
$$c) \qquad x + y - z = 2$$
$$x + 5y - 5z = 3$$

III: Consider the matrix N given by

$$N = \begin{bmatrix} 1, -1 \\ 1, -1 \end{bmatrix}$$

a) Compute all powers of N.
b) Compute all the positive powers of $A = I + N$ where I is the 2×2 idendity matrix.
c) Compute the inverse of the matrix $A = I + N$.
d) Compute also all the negative powers of A.
Consider the matrix N given by

$$N = \begin{bmatrix} 0, 1, 0 \\ 0, 0, 1 \\ 0, 0, 0 \end{bmatrix}$$

a) Compute all powers of N.
b) Compute all the powers of $A = I + N$ where I is the 3×3 idendity matrix.
IV:) Given are three vectors

$$\vec{a} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 3 \\ -7 \\ 5 \end{bmatrix} \quad \vec{c} = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}$$

a) Find a vector $\vec{n}$ normal to the plane spanned by the vectors $\vec{a}$ and $\vec{b}$.

b) Find the distance between the tip of vector $\vec{c}$ and the plane spanned by $\vec{a}$ and $\vec{b}$.

c) Find the volume ot the parallelepiped spanned by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

V: The transformation from the plane into itself consisting of rotating a vector by an angle θ and then stretching the resulting vector by a factor of λ, is a linear transformation. Find the associated matrix.