Test III for Calculus II, Math1502, March 14, 2000

Name:

    This test is to be taken without graphing calculators or notes of any sort. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\dots$. Please show all your work because otherwise credit cannot be given.

Vector or cross product:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - b_2 a_3 \\ a_3 b_1 - b_3 a_1 \\ a_1 b_2 - b_1 a_2 \end{bmatrix} \; .$$

I: The questions in this problem check your knowledge of some of the definitions and no partial credit will be given. Each question counts 5 points.

a) Add the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} , \quad \begin{bmatrix} 3 \\ 7 \\ -2 \end{bmatrix} .$$

b) Find the components of the vectors in the picture below.

c) Find $\vec{a} - \vec{b}$ where $\vec{a}$ and $\vec{b}$ are given in the picture below.

d) Find the angle between the two vectors

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} , \quad \begin{bmatrix} -c \\ d \\ a \\ -b \end{bmatrix} .$$

e) Which of the following two transformations from the plane to the plane is linear.

$$T(\vec{x}) = \vec{x} + \vec{a} \quad \text{where } \vec{a} \text{ is a fixed vector} .$$

$$T(\vec{x}) = \begin{bmatrix} \vec{a} \cdot \vec{x} \\ \vec{b} \cdot \vec{x} \end{bmatrix}$$

where $\vec{a}$ and $\vec{b}$ are two given vectors.

II: (30 points) Find **all** the solution of the following systems of equations. In each case state whether there is exactly one solution, no solution or infinitely many solutions. Use the row reduction technique. You do not have to bring the equation into echelon form.

$$a) \quad \begin{aligned} x - y + z &= 1 \\ x + y - z &= 2 \\ -x + y + z &= 3 \end{aligned}$$

$$b) \quad \begin{aligned} x - y + z &= 1 \\ x + y - z &= 2 \\ x + 5y - 5z &= 4 \end{aligned}$$

$$c) \quad \begin{aligned} x + 2y + 3z &= 6 \\ x + y &= 2 \\ y + 3z &= 2 \end{aligned}$$

III: (20 points) The expressions below define two linear transformations T_1 and T_2 from the plane into itself.

$$T_1 : \quad \begin{matrix} y_1 = 5x_1 + 3x_2 \\ y_2 = -x_1 + 2x_2 \end{matrix} \qquad T_2 : \quad \begin{matrix} y_1 = 3x_1 - x_2 \\ y_2 = -2x_1 + x_2 \end{matrix} \quad .$$

Find the matrices associated with T_1 and T_2. Find also the matrix associated with $T_1 \circ T_2$.

IV: (25 points) Given the three vectors.

$$\vec{a} = \begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix} \quad , \quad \vec{b} = \begin{bmatrix} -1/3 \\ 2/3 \\ -1/3 \end{bmatrix} \quad , \quad \vec{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

a) Find a vector $\vec{n}$ that is perpendicular to the plane spanned by $\vec{a}$ and $\vec{b}$.

b) Find the volume of the parallelepiped spanned by $\vec{a}$, $\vec{b}$ and $\vec{e_3}$.

c) Find the distance between the tip of the vector $\vec{a}$ and the vector $\vec{b}$.