TEST IV for Calculus II, Math1502, April 4, 2000

NAME:

This test is to be taken without graphing calculators, but a one–page summary of the relevant topics may be used. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Please show all your work because otherwise credit cannot be given.

I: (20 points) **(No partial credit can be given in this problem)**

a) For which vectors $\vec{a}$ and $\vec{b}$ in $\mathcal{R}^3$ are the vectors $\vec{a}$, $\vec{b}$ and $\vec{a} \times \vec{b}$ linearly independent?

b) True or false: The null space of a matrix A is never a subspace of the column space of that matrix.

c) True or false: The column space of a matrix A is always perpendicular to the null space of the transposed matrix A^T.

d) Consider the plane $x + y + z = 0$. Give a basis for this subspace of $\mathcal{R}^3$.

II: (20 points) Given the matrix

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 3 & \frac{3}{2} & \frac{9}{2} \\ 2 & 1 & 3 \end{bmatrix} .$$

Write down a basis for its column space and its null space.

III: (20 points) Find the vector $\vec{x}$ in $\mathcal{R}^3$ of smallest length that has its tip on the intersection of the planes $x + y + z = 1$ and $x - y - z = 2$.

$\vec{x} =$

IV: (20 points) Consider the matrix A and the vector $\vec{b}$.

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

a) Is $\vec{b}$ in the column space of the matrix A?

b) If not, compute the least square solution $\vec{x}^*$ of the system $A\vec{x} = \vec{b}$, i.e., find the vector $\vec{b}^*$ in the column space, closest to $\vec{b}$ such that $A\vec{x}^* = \vec{b}^*$.

$\vec{x}^* =$

$\vec{b}^* =$

V: (20 points) A linear transformation T has the following properties:

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right)=\begin{bmatrix}1\\0\\0\end{bmatrix},\quad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right)=\begin{bmatrix}0\\1\\0\end{bmatrix},\quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\\1\end{bmatrix}.$$

Find the matrix M_T associated with this transformation.

$M_T =$