Final Exam B, Calculus II, Math1502, December 10, 2001

Name:

This test is to be taken without graphing calculators and notes of any sorts, however, a cheat sheet, standard letter format, is allowed. The allowed time is 2 hours and 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations unless you are explicitely asked to do so! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Please show all your work because otherwise credit cannot be given.

Questions testing knowledge from Block 1

I: (15 points) Compute the integral

$$\int_0^{1/2} \frac{e^{y^2} - 1}{y} \mathrm{d}y$$

with two digits accuracy.

II: (15 points) a) Compute the limit

$$\lim_{x \to 0} \frac{1 - \cos(\sin(x^2))}{x^4} \ .$$

b) Find the 3-rd order Taylor polynomial (around 0) of the function

$$\ln(1 + \sin(x)) \ .$$

Questions testing knowledge from Block 2

III: (15 points) Find the interval of convergence, of the following power series. Include an analysis of what happens at the endpoints.

$$a) \quad \sum_{k=1}^{\infty} \frac{1}{k3^k}(x-3)^k \ .$$

$$b) \quad \sum_{k=0}^{\infty} \frac{k^k}{2^{k^2}} x^k \ .$$

IV: (15 points) Solve the differential equations

$$xy' + 3y = \frac{1}{x} \quad \text{with initial condition} \quad y(1) = 1$$

and

$$y' = \frac{y^2}{x^2} \quad \text{with initial condition} \quad y(1) = 2 \ .$$

Questions testing knowledge from Block 3

V: (15 points) Consider the following system of equations.

$$\begin{aligned} x + y - az &= 1 \\ 2x + y + 3z &= 2 \\ 2x + 3y + z &= b \end{aligned}$$

a) For which values of a and b is there a unique solution? For any of those values for a and b calculate this solution.

b) For which values of a and b, if any does this system have no solution?

c) For which values of a and b, if any does this system have infinitely many solutions?

VI: (5 points each, no partial credit) Find a one to one parametrization of the solutions of the following systems of equations. State for each of them which variables are pivotal.

$a)$
$$\begin{aligned} x - 2y + 3z &= 1 \\ y - 2z &= 2 \end{aligned}$$

$b)$
$$\begin{aligned} w + x + y + z &= 1 \\ y + z &= 0 \end{aligned}$$

$c)$
$$\begin{aligned} x + y + z &= 1 \\ y + z &= 0 \\ z &= 2 \end{aligned}$$

Questions testing knowledge from Block 4

VII: (15 points) Consider the matrix

$$A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 2 & 6 \\ 1 & 3 & 7 \end{bmatrix}$$

a) Find $\mathrm{Img}(A)$. Write its equation if it is a plane, give it in parametrized form if it is a line.

b) Find $\mathrm{Ker}(A)$. Write its equation if it is a plane, give it in parametrized form if it is a line.

VIII: (20 points) Find the QR decomposition of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 1 & 1 \\ 0 & 2 & 4 \end{bmatrix}$$

Questions testing knowledge from Block 5

IX: (20 points, no partial credit) True or False: a) The matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is diagonalizable.

b) Any matrix of the form $A^T A$ is diagonalizable.

c) Every matrix has real eigenvalues.

d) The matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is diagonalizable.

e) A projection matrix has only the eigenvalues one and zero.

f) Every matrix has an eigenvector.

X: (20 points) Compute the eigenvalues and eigenvectors of the following matrices. State which of the is diagonalizable and which is not.

a)

$$A = \begin{bmatrix} -2 & 4 \\ 4 & 4 \end{bmatrix}$$

b)

$$B = \begin{bmatrix} 7 & 5 \\ 8 & 4 \end{bmatrix}$$

c)

$$C = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$$

XI: (15 points) A 2×2 matrix A has the eigenvalues $\mu_1 = 9$, $\mu_2 = 1$ and the corresponding eigenvectors

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} .$$

a) Find U and D diagonal such that

$$A = UDU^{-1} .$$

b) Compute A.

c) Compute $\sqrt{A}$.

XII: (20 points) Find the solution of the system of differential equations

$$x^{'} = 8x + 4y \ , \ y^{'} = 4x + 2y$$

with initial condition $x(0) = 3$ and $y(0) = -1$.