Test I for Calculus II, Math 1502, September 12, 2000

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: (25 points)

a) Find the fourth order Taylor polynomial $P_4(x)$ for the function $\cos(x)$.

b) Using the above result, compute an approximate value for $\cos(1)$.

c) Give an estimate on how accurate the value computed in b) approximates $\cos(1)$.

II: (25 points) Compute the limits:

a)

$$\lim_{x \to 0} \frac{1 + x - e^x}{x(e^x - 1)}$$

b)

$$\lim_{n \to \infty} (n + e^n)^{1/n}$$

c)

$$\lim_{x \to 0} \frac{\cos(x) - \frac{1}{2}(e^x + e^{-x})}{x^2}$$

III: (25 points) Which of the following series is convergent or divergent.

a)

$$\sum_{k=1}^{\infty} k e^{-k^2} .$$

b)

$$\sum_{k=2}^{\infty} \frac{k}{k^2 - 1} .$$

c) Evaluate the series

$$\sum_{k=-2}^{\infty} \left(\frac{3}{4}\right)^k .$$

IV: (25 points) Decide which of the following improper integrals exists and compute its values if it exists:

a)

$$\int_0^2 \frac{1}{(1-x)^2}\mathrm{d}x$$

b)

$$\int_0^{1/2} \frac{1}{x\ln(x)}\mathrm{d}x$$

c)

$$\int_0^{\infty} e^{-x}\mathrm{d}x$$