Test I for Calculus II, Math 1502, September 11, 2001

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: (25 points)

a) Find the third order Taylor polynomial $P_3(x)$ and the remainder (for x close to zero) for the function

$$\frac{\sin(x)}{x} \ .$$

.

b) Using the above result, compute an approximate value for

$$A := \int_0^1 \frac{\sin(y)}{y} \mathrm{d}y \ .$$

c) Give an estimate on how accurate your value approximates A.

II: (25 points) Compute the limits:

a)

$$\lim_{x \to 0} \frac{\sin(x + x^3)}{x - x^3}$$

b)

$$\lim_{n \to \infty} (n^2 + n)^{1/n}$$

c)

$$\lim_{x \to 0} \frac{\sin(x + x^3) - x}{x^3}$$

III: (25 points) Which of the following series is convergent or divergent.

a)

$$\sum_{k=1}^{\infty} \frac{1}{k+k^2} \ .$$

b)

$$\sum_{k=2}^{\infty} \frac{1}{k \ln(k^2)} \ .$$

c) Evaluate the series

$$\sum_{k=1}^{\infty} \frac{3^{k-1}}{4^{k+2}} \ .$$

IV: (25 points) Decide which of the following improper integrals exists.

a)

$$\int_0^\infty \frac{1}{x+x^2}\mathrm{d}x$$

b)

$$\int_0^\infty \frac{1}{\sqrt{x}+x^2}\mathrm{d}x$$

c) Decide whether the following improper integral exists and if so, calculate its value.

$$\int_0^\infty \frac{e^x}{1+e^{2x}}\mathrm{d}x$$