Test II for Calculus II, Math1502, September 26, 2000

Name:

This test is to be taken without graphing calculators and notes of any sorts. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: (25 points) a) Given a sequence of positive numbers a_k with the property that $\lim_{k\to\infty} a_{k+1}/a_k = 1/2$. Does the series

$$\sum_{k=1}^{\infty} k a_k$$

converge or not?

b) With the same assumptions as in a), is the following series convergent?

$$\sum_{k=1}^{\infty} 3^k a_k \ .$$

c) Is the following series convergent?

$$\sum_{k=1}^{\infty} \left(\frac{k}{1+k}\right)^k \ .$$

II: (30 points) a) Find the smallest N so that $|L - s_N| \leq 10^{-2}$ where

$$L = \sum_{k=1}^{\infty} (-1)^k \frac{1}{k^k} \quad \text{and} \quad s_N = \sum_{k=1}^{N} (-1)^k \frac{1}{k^k} \ .$$

b) Find the interval of convergence of the series

$$\sum_{k=0}^{\infty} \frac{1}{[\ln(k)]^k} (x+2)^k \ .$$

c) Find the interval of convergence. Determine also whether the series converges at the endpoints or not.

$$\sum_{k=0}^{\infty} \frac{1}{k} x^k \ .$$

III: (30 points) a) Estimate to within 10^{-2} the integral

$$\int_0^1 \cos(x^2)\mathrm{d}x$$

b) Find the Taylor expansion around zero of the function

$$\frac{1}{(1-x)^2} \ .$$

c) Sum the following series exactly.

$$\sum_{k=0}^{\infty}(k+1)\left(\frac{2}{3}\right)^k \ .$$

IV: (15 points) Solve the following initial value problem

$$y' + \frac{5}{x}y = x^2$$

with the initial condition $y(1) = 2$