

Test II for Calculus II, Math 1502, September 25, 2001

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414....

I: (20 points) Check whether the following series converge or diverge.

a)

$$\sum_{k=1}^{\infty} \frac{k^2}{1+k^4} .$$

b)

$$\sum_{k=2}^{\infty} (-1)^k \frac{\ln(2k)}{(\ln(k))^2} .$$

c)

$$\sum_{n=0}^{\infty} \left(\frac{n}{1+n} \right)^{n^2} .$$

II: (30 points) Determine the interval of convergence of the following series.

a)

$$\sum_{k=0}^{\infty} \frac{(x+5)^k}{2^k}$$

b)

$$\sum_{k=0}^{\infty} \frac{k!}{(2k)!} x^k .$$

c)

$$\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k} \right)^4 x^k$$

III: (30 points)

a) Sum the series $\sum_{k=1}^{\infty} \frac{k}{10^k}$ exactly.

b) Find the Taylor series (around $x = 0$) of the function

$$\int_0^x \frac{e^t - 1}{t} dt .$$

c) Find the smallest N so that $|L - s_N| < 2^{-9}$, where

$$L = \sum_{k=0}^{\infty} \frac{1}{2^k}$$

and s_N is the N -th partial sum.

IV: A solution of brine with a concentration of 2 pounds of salt per gallon flows at a rate of 2 gallons per minute into a tank containing 1000 gallons of pure water. The well mixed solution flows out of the tank at the same rate of 2 gallons per minute.

a) (8 points) Set up the differential equation for the amount of salt in the tank at time t , call it $P(t)$.

b) (12 points) Solve the differential equation. What is the initial condition for $P(t)$?