Test III for Calculus II, Math1502, October 17, 2000

Name:

This test is to be taken without graphing calculators and notes of any sorts. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

Vector- or Cross product:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$$

I: The questions in this problem check your knowledge of some of the definitions and no partial credit will be given. Each question counts 5 points.

a) Add the vectors

$$\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -3 \\ -5 \\ -7 \end{bmatrix} .$$

b) Find the angle between the vectors

$$\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

c) Find the length of the vector

$$\begin{bmatrix} \sqrt{3} \\ \sqrt{6} \\ 4 \end{bmatrix}$$

d) Find the vector that when added to

$$\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \quad \text{yields} \quad \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} .$$

e) True or false: For two vectors $\vec{a}$ and $\vec{b}$, that have the same length, $\vec{a} - \vec{b}$ is always perpendicular to $\vec{a} + \vec{b}$.

II: (20 points) A linear transformation T of the plane into itself maps the vector

$$\vec{f}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{to} \quad \vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} ,$$

and the vector

$$\vec{f}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{to} \quad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

a) Find $T(\vec{f}_1 + \vec{f}_2)$, $T(\vec{e}_1)$, $T(\vec{f}_1 - \vec{f}_2)$ and $T(\vec{e}_2)$.

b) Find the matrix $[T]$ associated with this linear transformation.

III: (30 points) Solve the following systems of equations by row reduction:

a)

$$3x - y + 4z = 13$$
$$x + y + 2z = 9$$
$$3x + 3y + z = 12$$

b)

$$3x - 3y + z = -2$$
$$x - 2y + z = -3$$
$$2x - y = 1$$

c)

$$3x - 3y + z = 1$$
$$x - 2y + z = 1$$
$$2x - y = 1$$

IV: (25 points) a) Calculate the area of the parallelogram spanned by the vectors

$$\vec{a} = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} .$$

b) Calculate the volume of the parallelepiped that is spanned by the vectors $\vec{a}$, $\vec{b}$ and the vector $\vec{c}$.

$$\vec{c} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} .$$

c) Compute the distance between the tip of the vector $\vec{c}$ and the plane spanned by $\vec{a}$ and $\vec{b}$.

d) Find a vector $\vec{d}$ that is perpendicular to $\vec{a}$ and $\vec{b}$ such that the parallelepiped spanned by $\vec{a}$, $\vec{b}$ and $\vec{d}$ has the same volume as the one spanned by $\vec{a}$, $\vec{b}$ and $\vec{c}$.