Test III for Calculus II, Math 1502, October 23, 2001

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: No partial credit

a) (5 points) Which of the following transformations is linear?

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y-x \end{bmatrix}$$
$$g\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot y \\ x^2 - y^2 \end{bmatrix} .$$

b) (5 points) Compute the single vector given by

$$2\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} .$$

c) (5 points) Compute the length of the following vectors

$$\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} , \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} .$$

d) (5 points) Compute the angle between the following two vectors

$$\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} , \begin{bmatrix} -5 \\ -5 \\ 7 \end{bmatrix} .$$

e) (5 points) Find t such that

$$|\vec{a} + t\vec{b}|^2$$

is minimal, with

$$\vec{a} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$$

II: (20 points) Let f be a linear transformation from R^2 to R^2. Suppose that

$$f(\vec{e}_1 + \vec{e}_2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \ , f(\vec{e}_1 - \vec{e}_2) = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \ .$$

Find the matrix A_f corresponding to f.

b) (10 points) Find the distance between the tip of

$$\vec{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and the solution set of

$$2x_1 + 3x_2 - x_3 = 1 \ .$$

III: Give a one to one parametrization of the solutions of the following system of equations

$a)$ (5 points)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 2 \end{aligned},$$

Which of the variables are pivotal and which are non-pivotal?

$b)$ (10 points)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 2 \\ x_3 &= 3 \end{aligned},$$

Which of the variables are pivotal and which are non-pivotal?

IV: (30 points)

Find **all** the solution of the following systems of equations. In each case state whether there is exactly one solution, no solution or infinitely many solutions. Use the row reduction technique.

$$a) \quad \begin{aligned} x - y + z &= 2 \\ x + y - z &= 3 \\ -x + y + z &= 2 \end{aligned}$$

$$b) \quad \begin{aligned} x - y + z &= 0 \\ x + y - z &= 3 \\ x + 5y - 5z &= 9 \end{aligned}$$

$$c) \quad \begin{aligned} x + 2y + 3z &= 3 \\ x + y &= 2 \\ y + 3z &= -1 \end{aligned}$$