Solutions for Test 3

I) a)

$$\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

I) b) $\pi/2$.
I) c) 5
I) d)

$$\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$$

I) e) Always true.

II) a) $T(\vec{f_1} + \vec{f_2}) = T(\vec{f_1}) + T(\vec{f_2}) = \vec{e}_1 + \vec{e}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

But $\vec{f_1} + \vec{f_2} = 2\vec{e}_1$ and hence $T(\vec{e}_1) = 1/2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$T(\vec{f_1} - \vec{f_2}) = T(\vec{f_1}) - T(\vec{f_2}) = \vec{e}_1 - \vec{e}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

But $\vec{f_1} - \vec{f_2} = 2\vec{e}_2$ and hence $T(\vec{e}_2) = 1/2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

II) b) The associated matrix is given by

$$[T] = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

III) a) $x = 1, y = 2, z = 3$
III b) $x = -5/3 + z/3, y = 7/3 + 2z/3, z$ is arbitrary.
III) c) No solution

IV) a) Area $= \sqrt{50}$.
IV) b) Volume $= 2$
IV) c) Distance $= 2/\sqrt{50}$
IV) d)

$$\vec{d} = \pm \frac{2}{50} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$