Test IV for Calculus II, Math1502, November 7, 2000

Name:

This test is to be taken without graphing calculators. Notes of one single page are allowed. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: (25 points) Compute the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 3 & 4 & 6 \end{bmatrix}$$

II: (25 points) Find the least square solution for the system

$$A\vec{x} = \vec{b}$$

where

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 1 \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

III: (25 points) Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 5 & 1 & 7 \\ 7 & 1 & 10 \end{bmatrix} .$$

a) Find the column space of this matrix. Write the equation if it is a plane or the parametric representation if it is a line.

b) Find the null space. Write the equation if it is a plane or the parametric representation if it is a line.

c) The vector

$$\vec{x}_p = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

is a solution of $A\vec{x} = \vec{b}$ where

$$\vec{b} = \begin{bmatrix} 5 \\ 13 \\ 18 \end{bmatrix} .$$

Find all the solutions of $A\vec{x} = \vec{b}$ without using row reduction.

IV: (25 points) Consider all the vectors $\vec{x}$ in $\mathbf{R}^4$ that are perpendicular to the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} .$$

a) Do these vectors form a subspace? Give a reason.

b) If the answer to a) is yes, what is the dimension of this subspace?

c) If the answer to a) is yes, find a basis for this subspace.