Test IV for Calculus II, Math 1502, November 20, 2001

Name:

This test is to be taken without calculators, however, one sheet of notes (both sides, standard letter format) is allowed. The allowed time is 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: (No partial credit) True or false. For any $m \times n$ matrix A

a) (3 points) $\dim(\text{Ker}(A)) = \dim(\text{Ker}(A^T))$

b) (3 points) dim(Ker(A))+dim(Img(A))=n

c) (3 points) dim(Ker(A^T))+dim(Img(A^T))=n

d) (3 points) $\text{Ker}(A)^{\perp} = \text{Img}(A)$

e) (5 points) If $n \leq m$ then the column vectors of A are always linearly independent.

f) (3 points) The row rank equals the dimension of Img(A^T).

g) (5 points) True or false. The set of vectors in R^3, given by the equation $x + 2y + 3z = 1$ is a subspace of R^3?

II: Consider the orthonormal vectors

$$\vec{u}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_2 = \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}.$$

a) (10 points) The two vectors below span the same subspace as the vectors $\vec{u}_1$ and $\vec{u}_2$. Write each of them as a linear combination of $\vec{u}_1$ and $\vec{u}_2$.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}.$$

b) (5 points) Write the QR decomposition of the matrix $A = [\vec{v}_1, \vec{v}_2]$.

c) (10 points) Find the least square solution of the equation

$$A\vec{x} = \vec{b} \text{ where } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

III: Consider the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

a) (5 points) Compute Ker(A). Give its parametric representation if it is a line. If it is a plane, give its equation.

b) (5 points) Give a basis for Ker(A).

c) (5 points) Compute Img(A). Give its parametric representation if it is a line. If it is a plane, give its equation.

d) (5 points) Give a basis for Img(A).

IV: Consider the set S of all vectors in R^4 that satisfy the equation

$$w + x + y + z = 0$$

a) (5 points) Is this set a subspace of R^4?

b) (5 points) Find the orthogonal complement, $S^{\perp}$, of S, .

c) (10 points) Find the orthogonal projection matrix of R^4 onto $S^{\perp}$.

d) (5 points) Find the orthogonal projection matrix onto the space S.

e) (5 points) Find the distance between the subspace S and the tip of the vector

$$\vec{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$