Solutions for Test 4

I: a) False, b) True, c) False, d) False, e) False, f) True, g) False

II: a)

$$\vec{v}_1 = \sqrt{2}\vec{u}_1 \;\; \vec{v}_2 = (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1 + (\vec{u}_2 \cdot \vec{v}_2)\vec{u}_2 = \sqrt{2}\vec{u}_1 + 12/\sqrt{6}\vec{u}_2$$

b)

$$A = QR \text{ where } Q = [\vec{u}_1, \vec{u}_2] \text{ and } R = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 12/\sqrt{6} \end{bmatrix}$$

c) $R\vec{x} = Q^T\vec{b}$ and hence

$$\vec{x} = \begin{bmatrix} 13/12 \\ 5/12 \end{bmatrix}$$

III: a) $Ker(A)$ is given by all the multiples of the vector

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \tag{1}$$

b) The vector given in (1) is a basis for $Ker(A)$.
c) $Img(A)$ is given by the plane

$$x + y - 3z = 0 \; .$$

d) A basis for this plane is given by the vectors

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ \\ 1 \end{bmatrix}$$

IV: a) The given set is a subspace.
b) The orthogonal complement $S^\perp$ is given by all the multiples of the vector

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \; .$$

c)

$$P_{S^\perp} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

d) Since S and $S^\perp$ are complementary, we have that $P_S + P_{S^\perp} = I$ and hence

$$P_S = \frac{1}{4}\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}.$$

e) The distance is given by the length of the vector $P_{S^\perp}\vec{a}$. Now

$$P_{S^\perp}\vec{a} = \frac{1}{4}\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

whose length is one.