Final Exam for Calculus II, Math 1502, December 7, 2009

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Section:

Name of TA:

This exam is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Print your name (not your signature), your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

I abide by the honor code. **Signature:**

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Block 1:

1: (15 points) Calculate to three digits accuracy the integral $\int_0^1 e^{-x^2}\,dx$. You do not have to simplify the expression.

2: a) (7 points) Calculate the limit $\lim_{x\to 0} \frac{\log(1+x^2)-x^2}{x^4}$.

b) (8 points) Does the integral

$$\int_0^\infty \frac{\cos(\frac{1}{x})}{x^2}\,dx$$

exist? If it does, calculate it.

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Block 2:

3: a) (7 points) Find the interval of convergence, including the endpoints, of the power series $\sum_{k=0}^{\infty} \frac{(x-2)^k}{(k+1)^2}$.

b) (8 points) Calculate the value of the sequence $\sum_{k=1}^{\infty} \frac{k}{2^k}$.

4: (8 points each) Solve the initial value problems

$$a) \quad y' + 2y = 4 \ , \ y(0) = 3 \ .$$

$$b) \quad y' = \frac{y^2}{(1+x)^2} \ , \ y(0) = 1 \ .$$

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Block 3:

5: (12 points) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with

$$f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ and } f\left(\begin{bmatrix} 7 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} .$$

Find the matrix associated with f.

6: (10 points) Find all the solutions of the system

$$\begin{array}{cccc} x & +2y & +z & = 1 \\ 2x & & +3z & = 1 \\ -2x & +8y & -5z & = 1 \end{array} .$$

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Block 4:

7: (15 points) Find a basis for the kernel and the image of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 6 & 6 \\ 2 & 3 & 0 \end{bmatrix} .$$

Find also an equation describing Img(A).

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8: a) (15 points) Find the QR factorization of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 6 & 6 \\ 2 & 3 & 0 \end{bmatrix}$$

b) (5 points) Using the result of a) find the least square solutions for the equation $A\vec{x} = \vec{b}$ where $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

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Block 5:

9: True or False: (3 points each)
a) Every matrix is diagonalizable.
b) Every matrix whose eigenvalues have algebraic multiplicity one is diagonalizable.
c) The eigenvectors of a symmetric matrix can always be chosen to be perpendicular.
d) Two eigenvectors of a symmetric matrix may not be perpendicular.

10: (10 points) Calculate the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 6 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 4 & 3 \end{bmatrix} .$$

You do not have to calculate the eigenvectors. Is this matrix diagonalizable? Explain your answer.

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11: (15 points) Solve the system of differential equations

$$x' = 8x + 4y \ , \ \ y' = -3x + y \ ,$$

with the initial conditions $x(0) = -4, y(0) = 3$.

12: (15 points) Sketch the curve given by the equation

$$4x^2 + 6xy - 4y^2 = 5 \ ,$$

by carefully calculating the eigenvalues and the eigenvectors. What type of curve is it?