Test I for Calculus II, Math 1502, September 8, 2009

Name:

Section:

## Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean  $\sqrt{2}$  do not write 1.414.... Show your work, otherwise credit cannot be given.

Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

Section:

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**I:** (25 points)

a) Find the 6-th order Taylor polynomial  $P_6(x)$  for the function  $\exp(x^2)$ .

b) Using the above result, compute an approximate value for

$$\int_0^1 \frac{\exp(x^2) - 1 - x^2}{x^4} dx \; .$$

c) Give an estimate on how accurate the value computed in b) approximates the integral.

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**II:** (25 points) Compute the limits: a)

$$\lim_{x \to 0} \left(\frac{\sin(x)}{2x}\right)^2$$

b)

 $\lim_{n \to \infty} (1 + e^{-n})^{e^n}$ 

c)

$$\lim_{x \to 0} \frac{\int_0^x [e^{-y^2} - 1 + y^2] dy}{x^5}$$

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**III:** (25 points) Decide which of the following improper integrals exists and compute its values if it exists: a)

$$\int_0^\infty \frac{1}{\sqrt{1+x^2}} \mathrm{d}x$$

b)

$$\int_0^1 \frac{e^x}{\sqrt{e^x - 1}} \mathrm{d}x$$

c)

$$\int_0^\infty \frac{1}{x+x^{-1}} \mathrm{d}x$$

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**IV:** (25 points) Which of the following series is convergent or divergent. a)

$$\sum_{k=1}^{\infty} k e^{-k^2} \; .$$

b)

$$\sum_{k=2}^{\infty}rac{k}{k^2-1}\;.$$

c) Does the following series converge and if it does, evaluate it.

$$\sum_{k=2}^{\infty} 2^k e^{-k}$$