Test I for Calculus II, Math 1502, September 8, 2009

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

I: (25 points)

a) A function $f(x)$ satisfies the equation

$$f''(x) = xf(x) \ .$$

Moreover, it takes the values $f(0) = 0, f'(0) = 1$. Calculate the fourth order Taylor polynomial.

b) A function $f(x)$ has

$$P_3(x) = 1 - x + 2x^2 - 3x^3$$

as its third order Taylor polynomial. Write down the remainder term $R_3(x)$?

II: (25 points)

a) Find a function of the form $ax + b$ so that

$$\lim_{x\to\infty} \left[\sqrt{4x^2 + x - 1} - (ax + b)\right] = 0$$

b) A function $f(x)$ satisfies the equation $f'(x) = f(x)+1$ and $f(0) = 0$. Find

$$\lim_{x\to 0} \frac{f(x) - x}{x^2}$$

c)

III: (25 points) Decide which of the following improper integrals exists and compute its values if it exists: a)

$$\int_0^2 \frac{1}{(1-x)^2}\mathrm{d}x$$

b)

$$\int_0^{1/2} \frac{\sin(x)}{x^2\ln(x)}\mathrm{d}x$$

c)

$$\int_0^{\infty} xe^{-x}\mathrm{d}x$$

d) Extra credit:

$$\int_1^{\infty} \cos(x^2)dx$$

IV: (25 points) Which of the following series is convergent or divergent.

a)

$$\sum_{k=1}^{\infty} k3^{-k^2} \ .$$

b)

$$\sum_{k=2}^{\infty} \frac{k}{k^2-1} \ .$$

c) Does the following series converge and if yes what is its value?

$$\sum_{k=2}^{\infty} \frac{1}{k^2 - 1} \ .$$