Test III solutions for Calculus II, Math 1502, October 13, 2009

Print Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

Print your name (not your signature), your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

Print Name:

Section:

Name of TA:

I: a) (10 points) Given the matrices

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Calculate AB and BA.

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad BA = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}.$$

b) (5 points) What is the relation between the matrices A and B?
A is the left inverse of B or B is the right inverse of A.

c) (15 points) A linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^3$ has the property that

$$f\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \quad f\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Find the matrix A_f associated with the linear transformation f.

$$A_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

Print Name:

Section:

Name of TA:

II: a) (15 points) Find the length of the vectors

$$\vec{a} = \begin{bmatrix} 1 \\ -1 \\ \sqrt{2} \end{bmatrix} , \quad \vec{b} = \begin{bmatrix} 1 \\ 4 \\ -\sqrt{2} \end{bmatrix} ,$$

their dot product $\vec{a} \cdot \vec{b}$ as well as the distance between the tip of the vectors $\vec{a}$ and $\vec{b}$.

$$|\vec{a}| = 2 \ , |\vec{b}| = \sqrt{19} \ , \vec{a} \cdot \vec{b} = -5 \ , |\vec{a} - \vec{b}| = \sqrt{33}$$

b) (15 points) Find the angle between the vectors

$$\vec{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} .$$

$$\vec{a}\cdot\vec{b}=3\ ,\ \ |\vec{a}|=\sqrt{3}\ ,\ \ |\vec{b}|=2\ ,\ \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{\sqrt{3}}{2}$$

Angle is $\pi/6$.

Print Name:

Section:

Name of TA:

III: a) (10 points) Calculate the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} .$$

Check your answer!

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$$

b) (20 points) Given the vectors

$$\vec{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} ,$$

find the components of $\vec{a}$, $\vec{a}_{||}$ and $\vec{a}_{\perp}$, that are parallel and perpendicular to $\vec{b}$.

$$\vec{a}_{||} = \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{a}_{\perp} = \vec{a} - \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} .$$

Print Name:

Section:

Name of TA:

IV: a) (10 points) Which of the matrices below are isometries?

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & -2 \\ 1 & 2 \end{bmatrix}, \quad C = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & -2 \\ 2 & 1 \end{bmatrix}$$

A, C.

Extra Credit: (5 points) a) Find the 3×3 matrix that maps the vector $\vec{e}_1$ to $\vec{e}_2$, the vector $\vec{e}_2$ to $\vec{e}_3$, and the vector $\vec{e}_3$ to $\vec{e}_1$.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

b) (5 points) Is this matrix an isometry? YES, rotation by $2\pi/3$ about the axis in the direction

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} .$$