Final Exam, Calculus II, Math1502, May 4, 2000

Name:

This test is to be taken without graphing calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Write answers in boxes where provided. Provide exact answers; not decimal approximations unless you are explicitely asked to do so! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Please show all your work because otherwise credit cannot be given.

Questions testing knowledge from Block 1

I: (15 points) Compute the integral

$$\int_0^1 \frac{\sin(y)}{y}\,\mathrm{d}y$$

to three digits accuracy.

II: (15 points) a) Compute the limit

$$\lim_{x\to 0} \frac{\ln(1+x)+e^x-2x-1}{x^3}\ .$$

b) Find the 4-th Taylor polynomial (around 0) of the function

$$\ln(\cos(x))\ .$$

Questions testing knowledge from Block 2

III: (15 points) a) Does the following series converge ?

$$\sum_{k=1}^{\infty} \frac{(2k+1)^{2k}}{(5k^2+1)^k} \ .$$

b) Find the interval of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{k!}{k^k}(x-1)^k \ .$$

IV: (15 points) Solve the differential equations

$$xy' + 2y = \frac{1}{x} \quad \text{with initial condition} \quad y(1) = 2$$

and

$$y^2y' = x^2 \quad \text{with initial condition} \quad y(0) = 1 \ .$$

Questions testing knowledge from Block 3

V: (15 points) Find all the solutions of the following systems of equations.

$$a) \quad \begin{aligned} x + 3y - z &= 1 \\ 2x + 4y &= 1 \\ 3x + 5y + z &= 1 \end{aligned}$$

and

$$a) \quad \begin{aligned} 3x + 5y - 5z &= 1 \\ x - 7y + 33z &= 1 \\ -2x + y - 14z &= 2 \end{aligned}$$

VI: (15 points) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 2 & 2 & 1 \end{bmatrix}$$

Questions testing knowledge from Block 4

VII: (15 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ -3 & -9 & -6 \end{bmatrix}$$

Find a basis for the column space and a basis for the null space. Also, write these spaces in equation form.

VIII: (20 points) Find the solution of smallest length that satisfies the system of equations

$$\begin{aligned} x + 2z &= 1 \\ x + y - z &= 2 \end{aligned} .$$

Questions testing knowledge from Block 5

IX: (20 points) A 3×3 matrix A satisfies the equation

$$A^2 - A - 2I = 0$$

where I is the 3×3 identity matrix.

a) Find the possible eigenvalues.

b) Are there any repeated eigenvalues?

c) Sketch the curve defined by the quadratic form

$$x^2 + y^2 - 4xy = 1 \ .$$

X: (20 points) Compute the eigenvalues and eigenvectors of the following matrices:

a)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

b)

$$B = \begin{bmatrix} 10 & 2 \\ 8 & 4 \end{bmatrix}$$

c)

$$C = \begin{bmatrix} 10 & 2 \\ -8 & 2 \end{bmatrix}$$

XI: (15 points) A 2×2 matrix A has the eigenvalues $\lambda_1 = 1$, $\lambda_2 = -1$ and the corresponding eigenvectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} .$$

a) Find U and D diagonal such that

$$A = UDU^{-1} .$$

b) Compute A.

c) Compute A^{30}.

XII: (20 points) Find the solution of the system of differential equations

$$x' = 3x + 4y \ , \ y' = 4x - 3y$$

with initial condition $x(0) = 1$ and $y(0) = 0$.