Prepfinal, Calculus II, Math1502, May 1, 2000

Questions testing knowledge from Block 1

I: Compute the integral

$$\int_0^1 \frac{1-e^{y^2}}{y}\mathrm{d}y$$

to three digits accuracy.

II: Compute the limit

$$\lim_{x\to 0}\frac{\sin(x)-x\cos(x)}{x^3}$$

Find the 4-th Taylor polynomial (around 0) of the function

$$\sin(e^x)$$

Questions testing knowledge from Block 2

III: Does the following series converge ?

$$\sum_{k=1}^{\infty}\frac{k^{2k}}{k!}\ .$$

Find the interval of convergence of the power series

$$\sum_{k=0}^{\infty}\frac{8^k}{k!}x^k\ .$$

Compute exactly

$$\sum_{k=0}^{\infty}k^2\left(\frac{1}{2}\right)^k\ .$$

IV: Solve the differential equations

$$xy'-2y=x^3e^x$$

subject to the condition

$$y(1) = 0$$

and

$$y' = \frac{e^{x-y}}{1+e^x}$$

subject to the condition

$$y(1) = 0 \ .$$

Questions testing knowledge from Block 3

V: For which values of the parameters a and b has the follwing system of equations precisely one, none or infinitely many solutions?

$$\begin{aligned} x + 3y &= b \\ 2x + y - z &= 2 \\ x + 2y + az &= 1 \end{aligned}$$

VI: Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 4 \\ 1 & 4 & 0 \end{bmatrix}$$

Questions testing knowledge from Block 4

VII: Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 2 & 3 & 12 \end{bmatrix}$$

Find a basis for the column space and a basis for the null space. Also, write these spaces in parametrized form and in equation form.

VIII: Consider the equation

$$A\vec{x} = \vec{b}$$

where

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 2 & 3 & 12 \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Find the least square solution of this system. Is it overdetermined or underdetermined?

Questions testing knowledge from Block 5

IX: Sketch the curves defined by the quadratic forms

$$8x^2 + 2y^2 + 8xy = 1 \ ,$$

$$6x^2 + 8xy = 1$$

and

$$10x^2 + 4x^2 + 8xy = 1 \ .$$

X: Compute the eigenvalues and eigenvectors of the following matrices:

a)

$$A = \begin{bmatrix} 1 & a & b \\ 0 & 2 & c \\ 0 & 0 & 3 \end{bmatrix}$$

b)

$$B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 3 & 20 \end{bmatrix}$$

c)

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

XI: For the matrix B in b) above, compute $B^3 - 24B^2 + 65B - 42I$.

XII: Find all the solutions of the system of differential equations

$$x' = 4x + \sqrt{8}y \ , \ y' = \sqrt{8}x + 2y$$

Subject to the initial condition $x(0) = 1$ and $y(0) = 0$.