Practice Final for Calculus II, Math 1502, November 30, 2009

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

Block 1:

1: Calculate to three digits accuracy

$$\int_1^2 e^{\frac{1}{x}}\, dx \ .$$

2: a) Compute

$$\lim_{x \to 0} \frac{\ln(\cos x)}{x^2}$$

b) Calculate the integral provided it exists

$$\int_1^{\infty} \frac{\sin(\frac{1}{x})}{x^2} dx \ .$$

Block 2:

3: a) Find the interval of convergence (including the endpoints) of the power series

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} (x-1)^k \ .$$

b) Does the series

$$\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{k}$$

converge? If it does converge, give a reasonable estimate on n so that s_n, the n-th partial sum, and the limit differ by 10^{-5}.

4: Solve the differential equations

$$xy' + 5y = x^3 \ , \ y(1) = 1$$

and

$$y' = x(1 + y^2) \ , \ y(0) = 0 \ .$$

Block 3: Let f be a linear transformation from $\mathcal{R}^3$ to $\mathcal{R}^3$ such that

$$f\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \ , \ f\left(\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

and

$$f\left(\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \ .$$

Find the matrix associated with f.

5: Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 3 & 2 & 1 \end{bmatrix} \ .$$

Is there a vector $\vec{b} \in \mathcal{R}^3$ for which $A\vec{x} = \vec{b}$ has a solution? Either find it or explain why it does not exist.

Block 4:

7: Find the QR factorization of the matrix

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

and solve the least square problem $A\vec{x} = \vec{b}$ where

$$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} .$$

Is this least square solution unique? If not find the one that has least length.

8: Find a basis for the kernel and the image for the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 2 \end{bmatrix}$$

Give an equation for the image as well. Also, find the orthogonal projections onto Img(A) and Ker(A).

Block 5:

12: Sketch the curve defined by the equation

$$10x^2 + 8xy + 4y^2 = 12 .$$

13: Find the solution of the system of differential equations

$$x' = x + y \ , \ y' = -x + 3y$$

with the initial conditions $x(0) = 2$, $y(0) = 1$.

14: Diagonalize the matrix

$$\begin{bmatrix} 17 & -2 & -2 \\ -2 & 14 & -4 \\ -2 & -4 & 14 \end{bmatrix}$$

15: Solve, i.e., calculate a_n for all n, the finite difference equation

$$a_{n+1} = 3a_n + 4a_{n-1} \ , n = 0, 1, 2, \ldots$$

with the initial condition $a_0 = 1, a_1 = 1$.