

## Practice Test I for Calculus II, Math 1502, September 1, 2009

**Name:**

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean  $\sqrt{2}$  do not write 1.414.... Show your work, otherwise credit cannot be given.

**I:** (25 points)

a) Find the 12-th order Taylor polynomial  $P_{12}(x)$  for the function  $\cos(x^3)$ .

b) Using the above result, compute an approximate value for

$$\int_0^1 \frac{\cos(x^3) - 1}{x^3} dx .$$

c) Give an estimate on how accurate the value computed in b) approximates the integral.

d) A function  $f(x)$  satisfies the equation

$$f''(x) = xf(x) .$$

Moreover, it takes the values  $f(0) = 0, f'(0) = 1$ . Calculate the fourth order Taylor polynomial.

**II:** (25 points) Compute the limits:

a)

$$\lim_{a \rightarrow 0} \frac{a^x - 1 - \log(a)x}{x^2}$$

b)

$$\lim_{n \rightarrow \infty} (e^{2n} + e^n)^{1/n}$$

c)

$$\frac{\int_0^{\sin(x)} \sin(y^2) dy}{(e^x - e^{-x})^3}$$

**III:** (25 points) Decide which of the following improper integrals exists and compute its values if it exists: a)

$$\int_0^2 \frac{1}{(1-x)^2} dx$$

b)

$$\int_0^{1/2} \frac{\sin(x)}{x^2 \ln(x)} dx$$

c)

$$\int_0^\infty x e^{-x} dx$$

d) Extra credit:

$$\int_1^\infty \cos(x^2) dx$$

**IV:** (25 points) Which of the following series is convergent or divergent.

a)

$$\sum_{k=1}^{\infty} k e^{-k^2} .$$

b)

$$\sum_{k=2}^{\infty} \frac{k}{k^2 - 1} .$$

c) Evaluate the series

$$\sum_{k=-2}^{\infty} \left(\frac{3}{4}\right)^k .$$