Practice Test II for Calculus II, Math 1502, September 21, 2009

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

I: (25 points)

a) (7 points) Does the series

$$\sum_{k=2}^{\infty}(-1)^k \frac{1}{k \log k}$$

converge absolutely?

b) (10 points) Consider the series

$$\sum_{k=0}^{\infty}(-1)^k \frac{2^k}{(2k)!}$$

Does this series converge? If yes, calculate the first three digits after the decimal point of this limit.

c) (8 points) Let

$$\sum_{k} a_k x^k$$

be a power series and assume that it converges at $c > 0$. True or false:

1) The series converges for all $x < c$.
2) The radius of convergence, $r \geq c$.
3) The radius of convergence, $r \leq c$.
4) The series converges absolutely for x with $|x| < c$.

II: (25 points)

a) (8 points) Find the first three digits after the decimal point of

$$\int_0^1 \cos(x^2)dx \ .$$

b) (10 points) Sum the series

$$\sum_{k=2}^{\infty} k^2 \frac{1}{3^k}$$

c) (7 points) Find the power series for

$$\int_0^x \frac{\sin t}{t} dt$$

III: (25 points)

a) (7 points) Find the Taylor series for the function

$$\log(1 - x^2)$$

b) (10 points) Find the power series for $\tan^{-1}(x)$.

c) (8 points) Find the power series of $\sin(x)\cos(x)$.

IV: (25 points)

a) (7 points) Find the general solution of the differential equation

$$xy' + 2y = \frac{\cos x}{x}$$

b) (8 points) Solve the initial value problem

$$y' + xy = 1 \ , \ y(0) = 1 \ .$$

c) (10 points) A 1000 gallon tank is full of brine with a concentration of 50 grams per gallon. The mixture is emptied at a rate of 5 gallons pre minute and replenished (also at a rate of 5 gallons per minute) by a mixture that contains 30 grams per gallon. Find the amount of salt in the tank after t minutes.