Practice Test III for Calculus II, Math 1502, October 9, 2009

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

I: (15 points) Calculate all the possible matrix products of the matrices

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} , \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} , \quad C = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

II: (15 points) Consider a linear transformation $f : \mathcal{R}^2 \to \mathcal{R}^3$ with

$$f\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} , \quad f\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} .$$

Find the matrix A_f associated with f.

III: (10 points) a) Find the length of the vectors

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} , \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and their dot product.

IV: (15 points) Find the angle between the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

V: (10 points) Calculate the inverse of the matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

VI: (20 points) For the vector

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

find the components that is parallel respectively perpendicular to the vector

$$\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

VII: (15 points) Given the vector

$$\vec{a} = \frac{1}{5\sqrt{2}} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

a) find its length.
b) Find another vector $\vec{b}$ so that the matrix $[\vec{a}, \vec{b}]$ is an isometry.
Extra credit: (20 points) Find the distance from the tip of the vector

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

to the line $y = \frac{1}{2}x$.