

Practice Test IV for Calculus II, Math 1502, November 4, 2009

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414.... Show your work, otherwise credit cannot be given.

I: (15 points) The matrix A and the vector \vec{B} are given by

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 2 & a \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 1 \\ b \end{bmatrix}.$$

For which values of a, b is there no solution? For which values of a, b is there exactly one respectively infinitely many. Compute them.

II: (20 points) Using the normal equations, find the least square solutions of the system $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

III: (15 points) Are the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

linearly independent?

IV: (20 points) The matrix A is given by

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 4 & 2 & 6 & 0 \\ 4 & 3 & 1 & 2 \\ 1 & 2 & 0 & 3 \end{bmatrix}.$$

Find a basis for $\text{Img}(A)$ and for $\text{Ker}(A)$.

V: (15 points) Compute the inverse of

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & 2 & 0 \end{bmatrix} .$$

VI: a) (5 points) Given the basis in \mathcal{R}^3

$$\vec{u}_1 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} , \quad \vec{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} , \quad \vec{u}_3 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} ,$$

calculate the components of the vector

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

in this basis.

b) (10 points) The image of a $3 \times m$ matrix A is given by the equation $x + y + z = 0$. Find a basis for $\text{Ker}(A^T)$.