Practice Test IV B for Calculus II, Math 1502, November 7, 2009

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414.... Show your work, otherwise credit cannot be given.

I: (15 points) Using the Gram–Schmidt procedure for the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Find the orthogonal projection onto $\mathrm{Img}(A)$, $\mathrm{Ker}(A^T)$, $\mathrm{Img}(A^T)$ $\mathrm{Ker}(A)$.

II: (15 points) Using the LU factorization solve the equation $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix} , \quad \vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} .$$

Solve first $L\vec{y} = \vec{b}$, then $U\vec{x} = \vec{y}$ so that $LU\vec{x} = \vec{b}$. Check your answer.

III: (20 points) Compute the distance between the lines $\vec{x}_1(s) = \vec{x}_1 + s\vec{v}_1$ and $\vec{x}_2(t) = \vec{x}_2 + t\vec{v}_2$ where

$$\vec{x}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} , \quad \vec{x}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} , \quad \vec{v}_1 = \begin{bmatrix} 3 \\ -5 \\ -1 \end{bmatrix} , \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} .$$

IV: (15 points) Find the QR-factorization of the matrix

$$A = \begin{bmatrix} 1 & 5 & 9 & 5 \\ 2 & 4 & -3 & 1 \\ 2 & -2 & 3 & 1 \end{bmatrix} .$$

V: (15 points) The subspace S of $\mathcal{R}^4$ is spanned by the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \ , \quad \vec{v}_2 = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} \ , \quad \vec{v}_3 = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} \ .$$

Find the orthogonal projection onto S and the orthogonal projection onto $S^{\perp}$.

VI: (5 points) The $m \times n$ matrix A has rank r. What is dimKer(A), dimImg(A), dimKer(A^T), dimImg(A^T)?

Let S be a k-dimensional subspace of $\mathcal{R}^n$. True or false: (3 points each)

a) Any k vectors in S must be linearly independent.

b) Any k linearly independent vectors in S must be a spanning set.

c) Any $k+1$ vectors in S must be linearly dependent.

d) Any spanning set of S must consist at least of k vectors.

e) S_1, S_2 two subspaces of $\mathcal{R}^n$ then $S_1 \subset S_2$ implies $S_1^{\perp} \subset S_2^{\perp}$

f) S_1, S_2 two subspaces of $\mathcal{R}^n$ and $S_1 \cap S_2 \neq \vec{0}$, then $S_1^{\perp} \cap S_2^{\perp} \neq \vec{0}$.