Practice Final Exam for Calculus II, Math 1502, December 10, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

Problem	Score
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
XI	
XII	
Total	

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Problems related to Block 1:

I: (15 points) Compute with an error less than 10^{-3}

$$\int_2^3 e^{\frac{1}{x^2}}\,dx \ .$$

II: a) (7 points) Compute the limit

$$\lim_{x\to 0} \frac{e^x - \cos x - \sin x}{x^3}$$

b) (8 points) Does the improper integral

$$\int_0^1 \frac{1}{x^2} e^{\frac{1}{x}}\,dx$$

exist? If yes, compute it.

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Problems related to Block 2:

III: a) (7 points) Is the series

$$\sum_{k=0}^{\infty}(-1)^k \frac{(k!)^2}{k^{2k}}$$

convergent? Is it absolutely convergent?

b) (8 points) Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty}(-1)^k k^{-1+\frac{1}{k}}(x-2)^k$$

IV: (15 points) Solve the initial value problem

$$y' - \frac{1}{x^2}y = e^{-\frac{1}{x}} \ , \ y(1) = \frac{2}{e} \ .$$

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Problems related to Block 3:

V: (20 points) Consider the system of equations

$$2x + y + z = b$$
$$x + y - 2z = 2$$
$$x - y + az = -1$$

Determine all values for a and b for which this system has a) non solution, b) exactly one solution, c) infinitely many solutions. In the case b) and c) Compute all the solutions in terms of a and b.

VI: (15 points) A plane in $\mathbb{R}^3$ passes through the points

$$\mathbf{p_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} , \ \mathbf{p_2} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} , \ \mathbf{p_3} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

Give two representations of the plane, on in terms of parametrization and one in terms of an equation.

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VII: (20 points) Use the least square method to find the distance of the tip of the vector

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

to the plane given by

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Solve the problem in two ways, once using the normal equations and then using the QR factorization.

VIII: (15 points) Consider the matrix

$$\begin{bmatrix} 2 & 3 & 5 & 6 \\ 1 & 0 & 1 & 3 \\ 4 & 1 & 5 & 12 \\ 2 & 1 & 4 & 7 \end{bmatrix}$$

Find a basis for $Img(A)$ and for $Ker(A)$ as well as for $Img(A^T)$ and for $Ker(A^T)$. Try do this with a little computation as possible.

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IX: (15 points) Graph the curve given by the equation

$$11x^2 - 6xy + 19y^2 = 10 \ .$$

X: (15 points) Diagonalize the matrices

$$a) \quad \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix} \ .$$

$$b) \quad \begin{bmatrix} 6 & 9 \\ 4 & 11 \end{bmatrix}$$

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XI: (20 points) Solve the initial value problem given by the system

$$x' = 8x + 9y$$
$$y' = 4x + 13y$$
$$x(0) = 1 \ , \ y(0) = 2 \qquad\qquad (0.1)$$

Use both methods, the superposition principle and the exponentail of a matrix.

XII: (15 points) Solve the recursive relation, i.e., find a_n for arbitrary values of n,

$$a_{n+1} = 8a_n + 9a_{n-1}$$

with $a_0 = a_1 = 1$.