Practice Test 1B for Calculus II, Math 1502, September 10, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: (25 points) Using Taylor's theorem, calculate with and error less that 10^{-4} the integral

$$\int_0^1 \cos(x^4)dx \ .$$

Proceed as follows: a) Find the n-th order Taylor polynomial $P_n(x)$ for $\cos(x^4)$ and the remainder in Lagrange form.

b) Find n so that

$$|\int_0^1 P_n(x)dx - \int_0^1 \cos(x^4)dx| \leq 10^{-4} \ .$$

c) Compute the approximate value for the integral.

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II: (25 points) a) For what a does the limit

$$\lim_{x \to 0} \frac{\cos(x^2) - 1}{x^a}$$

exist and is not zero?

Compute: b)

$$\lim_{x \to 0} \frac{(1+x)^3 - 1 - 3x}{x^2}$$

c)

$$\lim_{x \to 0} \frac{\cos(\log(1+x)) - 1}{x^2}$$

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III: (25 points) a) Consider the integrals

$$a)\ \int_0^1 \frac{x}{\sqrt{1-x}}\mathrm{d}x \quad b)\ \int_0^\infty x\sin(x^2)\mathrm{d}x$$

Write down the definition what mean by *'this integral exists'* and then decide whether they indeed exist. Compute their values if they exist.

Use the comparison test to decide which of the following integrals exists:

$$c)\ \int_0^\infty \frac{1}{x+(x-1)^2}dx\ , \qquad d)\ \int_{-1}^\infty \frac{1}{1+x+x^2}\mathrm{d}x$$

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IV: (25 points) Which of the following series is convergent or divergent. Reason carefully! If the series is convergent sum it.
a)

$$\sum_{k=0}^{\infty}\left[\frac{1}{\sqrt{k+2}}-\frac{1}{\sqrt{k+1}}\right]$$

b)

$$\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+p)}\ ,$$

p a positive integer.

c) Consider the convergent series

$$L=\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)}$$

What is L? Find the smallest n so that $0 < L - s_n < 10^{-3}$.