Practice Test 3B for Calculus II, Math 1502, October 23, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: (25 points) Consider the linear transformation $f : \mathcal{R}^2 \to \mathcal{R}^3$ that maps the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ into the vector $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ and the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ into the vector $\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$. Likewise consider the linear transformation $g : \mathcal{R}^2 \to \mathcal{R}^2$ that maps the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ into the vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and the vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ into the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the matrices A_f, A_g and the matrix $A_{f \circ g}$. What about the matrix $A_{g \circ f}$?

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II: (25 points) a) Find the distance between the line

$$\vec{x}(t) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

and the origin. Hint: Calculate the distance between the point $\vec{x}(t)$ and the origin and minimize with respect to t.

b) For the vectors

$$\vec{a} = \frac{1}{\sqrt{30}} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \ , \ \vec{b} = \frac{1}{\sqrt{30}} \begin{bmatrix} 2 \\ -1 \\ -4 \\ 3 \end{bmatrix}$$

find a vector $\vec{c}$ such that the matrix

$$A = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$$

is an isometry.

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III: (25 points) a) Find a one-to-one parametrization of the solutions of the equations

$$x + 3y + 2z = 1 \ , \ 4x + 2y + z = 0 \ .$$

Do the same for

$$x + 3y + 2z = 0 \ , \ 4x + 2y + z = 1 \ .$$

b) Find all the right-inverses of the matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 2 & 1 \end{bmatrix} \ .$$

Does this matrix have a left inverse?

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IV: (25 points) a) For which values of A, B does the following system have exactly one, none or infinitely many solutions. Use row reduction.

$$x + 2y - 3z = 1 \ , \ 2x - 3y + 2z = 0 \ , -x + 12y + Az = B$$

b) Give a one-to-one parametrization of all the vectors that are perpendicular to the vector

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$