Test I for Calculus II, Math 1502 G1-G5 , September 14, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: (25 points) Consider the function $f(x) = \sqrt{4+x}$.
a) Find the 2nd order Taylor polynomial $P_2(x)$ for $f(x)$ and the corresponding remainder in Lagrange form.

b) Using the above result compute an approximate value, call it A, for $\sqrt{5}$

c) Give an estimate on how accurate the value computed in b) approximates $\sqrt{5}$, i.e., give a bound on

$$|\sqrt{5} - A| \ .$$

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II: (25 points) a) Let $f(x)$ be a continuous function on the real line. Compute

$$\lim_{x\to 0} \frac{\int_{-x}^{x} f(y)dy}{2x} \quad .$$

Compute as well: b)

$$\lim_{x\to 0} \frac{e^{(e^x)} - e}{x} \quad ,$$

c)

$$\lim_{x\to\infty} \frac{\sqrt{x^2+x+1}}{\sqrt{2x^2+1}} \quad .$$

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III: (25 points) Consider the integral

$$\int_0^\infty xe^{-x^2}\mathrm{d}x \ ,$$

Write down the definition what mean by *'this integral exists'* and then decide whether it indeed exist. Compute its value if it exists.

Similarly for

$$\int_2^\infty \frac{1}{x\log x}\mathrm{d}x$$

Using the comparison principle decide which of the two integrals below exist. State clearly if you use an upper bound or a lower bound in the comparison. You do not have to compute any of the integrals.

b)

$$\int_0^\infty \frac{1}{x+e^x}\mathrm{d}x$$

c)

$$\int_0^\infty \frac{1}{x+e^{-x}}\mathrm{d}x$$

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IV: (25 points) Which of the following series is convergent or divergent. If it is convergent, sum it.
a)

$$\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3)} \ .$$

b)

$$\sum_{k=0}^{\infty} \log \frac{k+2}{k+1} \ .$$

c) The following series converges

$$L = \sum_{k=2}^{\infty} \frac{2^k}{3^{k+1}}$$

Find L. Moreover, find the smallest n so that $0 < L - s_n < \left(\frac{2}{3}\right)^5$. Here s_n is the n-th partial sum.