**Test I R for Calculus II, Math 1502 G1-G5 , September 22, 2010**

**Name:**

**Section:**

**Name of TA:**

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

**Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

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**Name:**

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**I:** (25 points) Consider the function $e^{-x}$.
a) Find the 4-th order Taylor polynomial $P_4(x)$ for $e^{-x}$ and the corresponding remainder in Lagrange form.

$$P_4(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!}$$

and the remainder is

$$\frac{x^5 e^{-c(x)}}{5!}$$

where $c(x)$ is some number between 0 and $x$.
b) Using the above result compute an approximate value, call it $A$, for $\frac{1}{e}$
Set $x = 1$ and we get

$$A = P_4(1) = \frac{1}{2} - \frac{1}{3!} + \frac{1}{4!}$$

c) Give an estimate on how accurate the value computed in b) approximates $\frac{1}{e}$, i.e., give a bound on

$$|\frac{1}{e} - A| \ ,$$

using the remainder found in a).

$$|\frac{1}{e} - A| = \frac{e^{-c}}{5!}$$

where $c$ is some number between 0 and 1. Since

$$\frac{e^{-c}}{5!} \leq \frac{1}{5!} = \frac{1}{120}$$

and

$$|A - \frac{1}{e}| \leq \frac{1}{120} \ .$$

**Name:**

**Section:**

**Name of TA:**

**II:** (25 points) a) Let $f(x)$ be a continuous function on the real line. Compute

$$\lim_{x\to 0} \frac{\int_{-x}^{x} f(y)dy}{2x} .$$

$$f(0)$$

Compute as well: b)

$$\lim_{x\to 1} \frac{x-1}{x^5-1} ,$$

$$\frac{1}{5}$$

c)

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{4x}$$

$$e^4$$

**Name:**

**Section:**

**Name of TA:**

**III:** (25 points) Consider the integral

$$\int_0^\infty e^{-x/2} \mathrm{d}x \ ,$$

Write down the definition what mean by *'this integral exists'* and then decide whether it indeed exist. Compute its value if it exists.

Exists and equals 2.

Similarly for

$$\int_{-\infty}^\infty \frac{1}{x^2} \mathrm{d}x$$

Does not exist, because the integration extends through 0.

Using the comparison principle decide which of the two integrals below exist. State clearly if you use an upper bound or a lower bound in the comparison. You do not have to compute any of the integrals.

b)

$$\int_1^\infty \frac{1}{\sqrt{1+x^3}} \mathrm{d}x$$

exists using the comparison with the function

$$\frac{1}{x^{3/2}} \ .$$

c)

$$\int_1^\infty \frac{1}{(1+x^5)^{1/6}} \mathrm{d}x$$

does not exist since for $x \geq 1$ we have that

$$\frac{1}{(1+x^5)^{1/6}} \geq \frac{1}{(2x^5)^{1/6}} = \frac{1}{2^{1/6}x^{5/6}}$$

which is not integrable on $[1, \infty)$.

**Name:**

**Section:**

**Name of TA:**

**IV:** (25 points) Which of the following series is convergent or divergent. If it is convergent, sum it.
a)

$$\sum_{k=0}^{\infty} \left(1 + \frac{1}{k}\right)^{-k} .$$

does not converge since

$$\lim_{k \to \infty} \left(1 + \frac{1}{k}\right)^{-k} = \frac{1}{e} \neq 0$$

b)

$$\sum_{k=0}^{\infty} \frac{\sqrt{k+2} - \sqrt{k+1}}{\sqrt{k+1}\sqrt{k+2}} .$$

The partial sum is given by

$$\sum_{k=0}^{N} \frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+2}} = 1 - \frac{1}{\sqrt{N+2}} \to 1$$

c) The following series converges

$$L = \sum_{k=1}^{\infty} \frac{9}{10^k}$$

Find $L$. Moreover, find the smallest $n$ so that $0 < L - s_n < 10^{-10}$. Here $s_n$ is the $n$-th partial sum.

$$L = 1$$

$$s_n = \frac{9}{10} \sum_{k=1}^{n} \frac{1}{10^{k-1}} = \frac{9}{10} \sum_{k=0}^{n-1} \frac{1}{10^k} = \frac{9}{10} \frac{1 - 10^{-n}}{1 - 10^{-1}} = 1 - 10^{-n}$$

Hence

$$0 < 1 - s_n = 10^{-n}$$

and hence $n = 11$.