Test 4 for Calculus II, Math 1502, November 16, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

Problem	Score
I	
II	
III	
IV	
Total	

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I: The image of an $m \times n$ matrix A has the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

as a basis.

a) (3 points) What is m?

b) (5 points) What is the rank of the matrix A.

c) (7 points) What is the dimension of $Img(A^T)$ and $Ker(A^T)$?

d) (10 points) Find a basis for $Ker(A^T)$.

e) (10 points) Give an equation for $Img(A)$.

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II: A matrix A has a QR factorization where $Q = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & -2 \end{bmatrix}$, $R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$.

a) (10 points) Find the orthogonal projections onto $Img(A)$ and onto $Ker(A^T)$.

b) (5 points) Find the vector in $Img(A)$ that is closest to $\vec{b}$, where $\vec{b} = \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix}$.

c) (5 points) Find the distance between the tips of the vector found in b) and the vector $\vec{b}$.

d) (10 points) Find all the solutions of the least square problem $A\vec{x} = \vec{b}$.

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III: Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 12 \\ 2 \end{bmatrix}$.

a) (10 points) are these vectors linearly independent?

b) (5 points) Give a basis for the space spanned by the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

c) (10 points) Suppose a 4×3 matrix has rank 2. Then

$$dim(Img(A)) =$$

$$dim(Ker(A)) =$$

$$dim(Img(A^T) =$$

$$dim(Ker(A^T)) =$$

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IV: Consider the matrix

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 5 & 4 \\ 6 & 2 & -2 \end{bmatrix}$$

a) (10 points) Apply the Gram-Schmidt procedure to calculate an orthogonal basis for $Img(A)$.

b) **Extra Credit:** (10 points) Calculate the QR factorization of the matrix A.