Practice Test 2 for Anaysis I, Math 4317.

1: Problem 32 on page 94 of IA.
2 : Problem 34 on page 94 of IA.
3 : Let $C \subset E^n$ be a closed set. Let $x_0 \in E^n$ be a point. Recall that the distance from x_0 to C is given by

$$d(x_0, C) := g.l.b\{|x_0 - y| : y \in C\} \ .$$

Show that there exists $y \in C$ such that

$$d(x_0, C) = |x_0 - y| \ .$$

Is this point unique? Give examples.
4 : Let C and D be two closed subsets of E^n and define the distance between these sets by

$$d(C, D) = g.l.b.\{|x - y| : x \in C, y \in D\} \ .$$

If one of these sets is compact, then there exists $x_0 \in C$ and $y_0 \in D$ such that

$$|x_0 - y_0| = d(C, D) \ .$$

Is the compactness condition really needed?
5 : Recall that for a set $S \in E$ where E is a metric space, the interior of S is defined by

$$S^\circ := \cup_{U \subset S, open} U$$

and the closure of S by

$$\overline{S} := \cap_{F \supset S, closed} F$$

Finally consider the boundary of S

$$\partial S = \overline{S} \cap \overline{S^c} \ .$$

Prove or disprove:
a) We have always

$$\overline{S^\circ} = \overline{S} \ .$$

b) There exists a metric space and a set $S \subset E$ such that the boundary of the set S is S.
c) We have always

$$\overline{S}^\circ \neq \emptyset$$

d)

$$E = S^\circ \cup \partial S \cup (S^c)^\circ$$

where all the sets on the right are disjoint.
6 : Let $p_1, p_2, \ldots$ be a convergent sequence in a metric space E with limit p. Show, by applying the definition that the set $\{p, p_1, p_2, \ldots\}$ is compact.
7 : Is the function $x^{1/3}$, defined for all real numbers, uniformly continuous?
8 : Find the integer part of the root of $x^3 - x + 1$.
9 : Problem 10 on page 92 of IA