Test 1 for Anaysis I, Math 4317, September 24, 2008, Allowed time 50 minutes. This is a closed book test.

Always state your reasoning otherwise credit will not be given. Try to be as concise and to the point as possible.

Name:

1: (10 points) For any two subsets of S show that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

2 :(20 points) Consider the sequence given recursively by $x_0 = 1$ and

$$x_{n+1} = 1 + \frac{1}{2} x_n \ , \ n = 1, 2, \cdots$$

Is the sequence monotone? Is it bounded? Does this sequence converge? If yes what is its limit?

3: (15 points) Let A be an open subset, and B be a closed subset of a metric space E. Show that the complement of $A \cap B$ in A is an open set.

4: (30 points) Prove that every Cauchy Sequence in the set of real numbers has a limit in the real numbers.

5: True or false: (each item counts 5 points)

a) Every compact metric space is bounded and closed.

b) Any subset of a compact set is compact.

c) In any metric space bounded and closed subsets are compact.

d) Every compact metric space is complete.

e) A closed subset of a complete metric space is closed.