Problem 5 Here is one way to understand this problem.

Suppose that I_α, $\alpha \in A$ is a collection of open intervals. We claim that there exists a collection of open intervals J_β $\beta \in B$ that are disjoint such that

$$\bigcup_{\alpha \in A} I_\alpha = \bigcup_{\beta \in B} J_\beta$$

Clearly two open intervals that intersect can be fused into a single open interval.

Pick $\gamma_1, \gamma_2 \in A$. We say that γ_1 and γ_2 are equivalent if there ex. $C \subset A$ with $\gamma_1, \gamma_2 \in C$ and

$$\bigcup_{\alpha \in C} I_\alpha \quad \text{is an open interval.}$$

We write $\gamma_1 \sim \gamma_2$. Clearly if $\gamma_1 \sim \gamma_2$ we have that $\gamma_2 \sim \gamma_1$. Of course $\gamma_1 \sim \gamma_1$. Suppose that $\gamma_1, \gamma_2, \gamma_3 \in A$ and

$\gamma_1 \sim \gamma_2$ and $\gamma_2 \sim \gamma_3$. We claim that $\gamma_1 \sim \gamma_3$. To see this we note that there ex. $C_1 \subset A$ with $\gamma_1, \gamma_2 \in C_1$ and

$$K_1 = \bigcup_{\alpha \in C_1} I_\alpha \text{ is an open interval}$$

Likewise there ex. $C_2 \subset A$ with $\gamma_2, \gamma_3 \in C_2$ and

$$K_2 = \bigcup_{\alpha \in C_2} I_\alpha \text{ is an open interval}$$

Now we consider $C = C_1 \cup C_2$. We have that $\gamma_1, \gamma_3 \in C_1 \cup C_2 = C$. Further

$$K_1 \cup K_2 = \bigcup_{\alpha \in C} I_\alpha \text{ and since}$$

$I_{\beta_2} \subset K_1$ and $I_{\beta_2} \subset K_2$ and since

K_1 and K_2 are open interval $K_1 \cup K_2$ is also an open interval. Thus $\gamma_1 \sim \gamma_3$.

~~Pick $\gamma \in A$ and consider $C_\gamma = \{$ [illegible] $\}$~~

For $\gamma \in A$ define $C_\gamma = \{\alpha \in A : \alpha \sim \gamma\}$.

We claim that for $\gamma_1, \gamma_2 \in A$ either

$C_{\gamma_1} \cap C_{\gamma_2} = \phi$ or $C_{\gamma_1} = C_{\gamma_2}$.

This is clear since if $\gamma \in C_{\gamma_1} \cap C_{\gamma_2}$ we have that $\gamma_1 \sim \gamma$ and $\gamma \sim \gamma_2$ and thus $\gamma_1 \sim \gamma_2$ and hence $C_{\gamma_1} = C_{\gamma_2}$.

If $C_{\gamma_1} \cap C_{\gamma_2} = \phi$ then

$$K_{\gamma_1} = \bigcup_{\alpha \in C_{\gamma_1}} I_\alpha \quad \text{and} \quad K_{\gamma_2} = \bigcup_{\alpha \in C_{\gamma_2}} I_\alpha$$

are both open intervals that are disjoint.

If $C_{\gamma_1} = C_{\gamma_2}$ then $K_{\gamma_1} = K_{\gamma_2}$. Thus

$$\bigcup_{\gamma \in A} K_\gamma = \bigcup_{\alpha \in A} I_\alpha$$

and the open intervals K_γ are either disjoint or identical. |

Suppose that S is an open bounded set in ~~the~~ the real numbers.
For any $x \in S$ there ex. $\varepsilon_x > 0$ that

$I_x = (x - \varepsilon_x, x + \varepsilon_x) \subset S$. Hence

$$S = \bigcup_{x \in S} I_x$$ where I_x are open intervals. By the previous considerations these union can be written as a union of disjoint intervals.