Practice Test 1 for Anaysis I, Math 4317, September 23, 2010

Always state your reasoning otherwise credit will not be given

1: For any two subsets of S show that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$$

2 : Let X be a finite set and $f : X \to X$ be a function that is one-one. Show that f is onto.

3 : Consider the sequence given recursively by $x_0 = 3$ and

$$x_{n+1} = \frac{1}{3}(x_n + x_n^2)$$

Does this sequence converge?

4: Consider the sequence

$$a_n = \frac{2n^2 + 3}{n^2 + n + 1} \ .$$

Give a rigorous proof that a_n converges to 2.

5: Prove that if a sequence $a_1, a_2, \cdots$ of real numbers converges to some number a, then the sequence

$$b_n := \frac{\sum_{k=1}^{n} k a_k}{n^2}$$

also converges. What is the limit of this sequence? Is the converse true?

6: Find a complete metric space and a sequence of bounded closed sets $S_i, i = 1, 2, \cdots$ such that

$$S_1 \supset S_2 \supset S_3 \cdots$$

but

$$\cap_{i=1}^{\infty} S_i = \emptyset$$

7: Prove that every bounded monotone sequence of real numbers converges.

8 : Let A and B be subsets of a metric space. Assume that A is closed and B is open. Show that the complement of $A \cap B$ is closed as a subset of the metric space A.

9 : Find a collection of nonempty closed subsets of the real numbers whose union is bounded and open.

10: Is the set consisting of all rational numbers r with $0 \le r \le 1$ a compact subset of the real numbers?