Test 1 for Anaysis I, Math 4317, September 29, 2010, Allowed time is 50 minutes. This is a closed book test.

Always state your reasoning otherwise credit will not be given. Try to be as concise and to the point as possible.

Name:

1: (10 points) Consider the subset S of the real numbers given by

$$S = (-1, 1) \cup \{2\} .$$

a) Find the closure $\overline{S}$, i.e., the smallest closed set that contains S.

b) Find the open interior of S, i.e., the largest open subset of S.

2 :(15 points) For which real numbers x is the following inequality true:
a) $x^2 + 2x - 3 < 0$

b) Show that the x axis is a closed subset of the plane.

3 :(15 points) Consider the sequence given recursively by $x_0 = 1$ and

$$x_{n+1} = \frac{1}{3}(x_n + x_n^2) \ , \ n = 1, 2, \cdots$$

Is the sequence monotone? Is it bounded? Does this sequence converge? If it does, what is its limit?

4: (15 points) Consider the subset S of the real numbers consisting of all series of the form

$$\sum_{k=1}^{\infty} \frac{a_k}{2^k}$$

where $a_k \in \{0, 1\}$. What is the least upper bound of the set S?

5: (20 points) Let E be a compact metric space. Prove that if $S_i, i \in I$ is a collection of closed subsets of E such that for any finite sub-collection $J \subset I$

$$\cap_{i \in J} S_i \neq \emptyset \ ,$$

then

$$\cap_{i \in I} S_i \neq \emptyset \ .$$

Hint: Reformulate the problem by taking complements in E.

6: Let E be a metric space. True or false: (each item counts 5 points)

a) The intersection of an arbitrary collection of closed subsets of E is closed.

b) Any closed subset of a compact subset of E is compact.

c) Bounded and closed subsets of E are compact.

d) Every compact metric space is complete.

e) A subset of a complete metric space is complete.