Final Exam A for Calculus II, K1-K6, Math 1502, December 10, 2013

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

Problem	Score
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
Total	

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I: a) (10 points) Write the following integral in terms of an alternating series,

$$\int_0^1 e^{-x^2} dx \ .$$

b) (5 points) Determine the smallest n so that the partial sum s_n of the series obtained in part a) approximates the integral with an error less than 10^{-3}.

II: a) (7 points) Compute the eighth order Taylor polynomial (at $a = 0$) of the function

$$\cos(x^2)$$

b) (8 points) For which which values of $\alpha > 0$ does the integral

$$\int_2^\infty \frac{1}{x(\ln x)^\alpha} dx$$

exist? Compute it for these values of α.

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III: a) (7 points) Compute the interval of convergence of the series

$$\sum_{k=1}^{\infty} \frac{2^k}{k}(x-1)^k \ .$$

State the convergence test you are using.

b) (8 points) Does the series

$$\sum_{n=1}^{\infty} \frac{n^n}{3^{n \ln n}}$$

converge? State the convergence test you are using.

IV: (15 points) Find the solution of the initial value problem

$$\frac{dx}{dt} = \frac{x^2}{t} \ , \ x(1) = 1 \ .$$

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V: Consider the three vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ a \end{bmatrix}.$$

a) (8 points) Determine all the values for a and b for which the vector

$$\vec{b} = \begin{bmatrix} b \\ 2 \\ -1 \end{bmatrix}$$

is in the span of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

b) (7 points) Determine all the values for a and b for which the vector $\vec{b}$ is a unique linear combination of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

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VI: a) (7 points) Using the normal equations find the least square solution of the inconsistent system $A\vec{x} = \vec{b}$

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$

b) (7 points) The matrix B has the QR factorization $B = QR$ where

$$Q = \frac{1}{9}\begin{bmatrix} 1 & 8 \\ 4 & -4 \\ 8 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 9 & 9 \\ 0 & 18 \end{bmatrix}, \quad \vec{c} = \begin{bmatrix} 13 \\ 7 \\ 5 \end{bmatrix}$$

Solve the least square problem $B\vec{x} = \vec{c}$.

c) (6 points) Find the projection of $\vec{c}$ onto the column space of B.

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VII: (8 points) a) Find a basis for the column space and a basis for the null space of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 7 \\ -1 & 0 & 1 & -1 \\ 2 & 1 & 0 & 5 \\ 3 & 1 & 2 & 12 \end{bmatrix}.$$

b) (7 points) What are $rank(A^T)$ and $dimNul(A^T)$?

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VIII: a) (7 points) Find all the eigenvalues of the matrix

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 4 \\ 1 & 0 & 4 \end{bmatrix} .$$

(You do not have to find the eigenvectors).

b) (3 points) Is B diagonalizable?

c) (10 points) The matrix

$$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

has the eigenvalues 7 and -2. Find an orthonormal basis of eigenvectors for A.

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IX: Consider the curve given by the equation

$$-17x^2 + 18xy + 7y^2 = 10 \ .$$

a) (3 points) Find the associated 2×2 matrix.

b) (5 points) Find the eigenvalues and normalized eigenvectors

c) (2 points) What is the type of the curve, hyperbola or ellipse?

d) (5 points) Sketch this curve below.

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X: True or false? You do not have to give a reason. Each problem counts 3 points.

a) A 3×5 matrix has a null space whose dimension is greater or equal than 2.

b) A 5×3 matrix has a column space whose dimension is 2. Then the dimension of the null space of A is 1

c) The eigenvectors of a symmetric square matrix do necessarily form an orthonormal basis.

d) A 2×2 matrix satisfies the equation (as matrices) $A^2 = 4I$. Then, necessarily, A must have both, 2 and -2 as eigenvalues.

e) A 5×3 matrix has a column space whose dimension is 2. Then the dimension of the null space of A^T is 3