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I: (25 points) Calculate the limits:

a)

$$\lim_{x \rightarrow 0} \frac{f(x)}{f^{-1}(x)},$$

where $f(x)$ is a differentiable and invertible function with $f(0) = 0$ and $f'(0) = 4$.

b)

$$\lim_{x \rightarrow 0} \frac{x - \int_0^x [\cos(t)]^2 dt}{x^3}$$

c)

$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin(2x)} - \frac{1}{\tan(2x)} \right)$$

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II:(25 points) a) Decide which of the following improper integrals exists and compute its values if it exists:

$$a) \int_0^{\infty} e^{-x} \cos(x) dx, \quad b) \int_0^{\infty} \frac{x}{1+x^2} dx$$

Use the comparison test to decide which of the following integrals exists:

$$c) \int_0^{\infty} \frac{1}{[\sin(x)]^2 + x^2} dx, \quad d) \int_0^{\infty} \frac{x^2}{\sqrt{1+x^6}} dx$$

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III: (25 points) Which of the following series is convergent or divergent. Reason carefully!

a)

$$\sum_{k=1}^{\infty} \left(\frac{k+1}{k} \right)^{k^2}$$

b)

$$\sum_{k=0}^{\infty} \frac{1}{(k+2)(k+3)} .$$

c) Consider the convergent series

$$L = \sum_{k=0}^{\infty} \frac{1}{3^k}$$

Find the smallest n so that $0 < L - s_n < 10^{-3}$.

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IV: a) Solve the differential equation

$$\frac{dx}{dt} = \sin x$$

with initial condition $x(0) = \pi/2$.

b) At a certain moment, a tank contains 100 liters of brine with a concentration 40 grams of salt per liter. The brine is continuously drawn off at a rate of 10 liters per minute and replaced by brine containing 20 grams salt per liter. Find the amount of salt in the tank at time t later.