Test 4 for Calculus II, Math 1502 K1 - K6, November 13, 2013

PRINT Name:

PRINT Section:

PRINT Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

PRINT your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: a) (12 points) Find a basis for the Null Space of the matrix

$$A = \begin{bmatrix} 1 & 1 & -3 & -1 \\ 3 & 0 & 6 & 9 \\ 0 & 1 & -5 & -4 \end{bmatrix}$$

b) (2 points) What is the dimension of $Nul(A)$?

c) (6 points) What is the dimension of $Col(A)$?

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II: a) (10 points) Given the vectors

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{c} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

compute the volume of the parallelepiped spanned by $\vec{a}, \vec{b}, \vec{c}$.

b) (2 points) Are the vectors $\vec{a}, \vec{b}, \vec{c}$ linearly independent?

c) (8 points) With the least amount of computation calculate the determinant of the matrix

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

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III: Given the matrices

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \ , \ U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

a) (5 points) Show that the matrix $A = LU$ is invertible.

b) (5 points) Compute the inverse of L.

c) (5 points) Compute the inverse of U.

d) (5 points) Compute the inverse A^{-1}.

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IV: a) (8 points) Find the eigenvalues and the eigenvectors of the matrix

$$A = \begin{bmatrix} 7 & 8 \\ 2 & 1 \end{bmatrix}$$

b) (4 points) Find the eigenvectors and eigenvalues for A^2.

c) (8 points) Find the eigenvalues and eigenvectors of the matrix

$$B = \begin{bmatrix} 13 & 16 \\ -9 & -11 \end{bmatrix}$$

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V: No partial credit: (5 points each) True or false (you do not have to give a reason):

a) Given a matrix A which is not invertible and a vector $\vec{b}$. Then $A\vec{x} = \vec{b}$ has no solution.

b) An $n \times n$ matrix has n distinct eigenvalues. Then the eigenvectors are linearly independent.

c) The collection of vectors $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ that satisfy the equation $x+y+z=1$ form a subspace of $\mathcal{R}^3$.

d) Let A be an $n \times n$ matrix. The matrix B obtain by adding a multiple of one row of A to another row of A does not change the eigenvalues.