Homework 4, due Wednesday April 5

I: Let x be an element of a Banach algebra $\mathcal{A}$ with in involution $x \to x^*$. Assume that for all $x \in \mathcal{A}$, $\|x\|^2 \leq \|xx^*\|$. Show that $\mathcal{A}$ is a C^* algebra.

II: Consider the set $\mathcal{A}$ of continuous complex valued functions on the closed disk $D = \{z : |z| \leq 1\}$ in the complex plane that are analytic in the interior of D. Let $\|f\| = \sup_{|z|\leq 1} |f(z)|$. Show that $\mathcal{A}$ is a commutative Banach algebra with unit. (Hint: Cauchy's theorem might come in handy).

III: With the same setup as in the previous problem, show that $f^*(z) = \overline{f(\overline{z})}$ defines an isometric involution in $\mathcal{A}$.

IV: Let $\mathcal{H}$ be a Hilbert space and $L(\mathcal{H})$ with the operator norm is the space of bounded operators. Show that $L(\mathcal{H})$ is a (non-commutative) C* algebra.

V : Show that the space of compact operators form a closed subalgebra without unit of $L(\mathcal{H})$.