PRACTICE FINAL EXAM

1. Curves

Problem 1: Find the parametric equations of the line that is tangent to the curve

$$\vec{r}(t) = (e^t, \sin t, \ln(1-t))$$

at $t = 0$.

Problem 2: Find the speed and the normal and tangential components of the acceleration and curvature for the curve $x(t) = \cos t, y(t) = \sin(t), z(t) = -t^2$.

2. Optimization problems

Problem 3: Find the minimum cost area of a rectangular solid with volume 64 cubic inches, given that the top and sides cost 4 cents per square inch and the bottom costs 7 cents per square inch. Just set up the equations using Lagrange multipliers, you do not have to solve them.

Problem 4: Find the plane of the form

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

where $a, b, c > 0$ and that passes through the point $(2, 1, 4)$ and cuts off the smallest volume in the first octant.

3. Double and triple integrals

Problem 5: Find the y moment of the first petal (mostly in the first quadrant) of the 3-leaf rose $r = \cos(3\theta)$. Just set up the integral (with limits) in polar coordinates. You do not have to evaluate it.

Problem 6: Compute the volume of the region that is bounded above by the plane $z = y$ and below by the paraboloid $z = x^2 + y^2$.

4. Surface Integrals

Problem 7: Find the surface area of the parabolic cylinder $z = y^2$ that lies over the triangle with vertices $(0,0), (0,1), (1,1)$ in the zy plane.

Problem 8: Consider the surface $x^2 + y^2 + (z-2)^2 = 4, z \geq 0$. Convert via Stokes' theorem the surface integral

$$\int_S \int \text{curl} F \cdot \vec{n} d\sigma$$

to a line integral. Here $\vec{F} = x^2 y\vec{i} - xy^2\vec{j} + \sin z\vec{k}$. Set this line integral up, parametrize the curve, and reduce to an ordinary Calculus One integral with limits. Don't evaluate this integral.

5. Line integrals and Stokes' Theorem

Problem 9: Compute the line integral of the vector field

$$\vec{F} = (xyz + 1, x^2 z, x^2 y)e^{xyz}$$

along the curve given in parametrized form by

$$\vec{r}(t) = (\cos t, \sin t, t) \ , \ 0 \leq t \leq \pi \ .$$

Problem 10: Compute the line integral $\int_C \vec{F} \cdot \vec{dr}$ where C is the curve given by the intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the plane $z = -y$, counterclockwise when viewed from above, and

$$\vec{F} = (x^2 + y, x + y, 4y^2 - z) \ .$$

6. Divergence Theorem

Problem 11: Use the divergence theorem to compute the outward flux of the vector field

$$\vec{F} = (x^2, y^2, z^2)$$

through the cylindrical can that is bounded on the side by the cylinder $x^2 + y^2 = 4$, bounded above by $z = 1$ and below by $z = 0$.

Problem 12: Compute the flux of $\vec{F} = 5zy^3\vec{i} + xz\vec{j} + 3z\vec{k}$ through the surface $x^2 + y^2 + z^2 = 9$ using the divergence theorem.